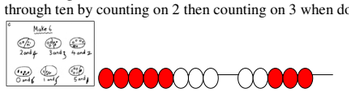
PROBLEM SOLVING

FLUENCY REASONING

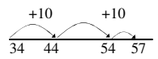
Progression in Addition

Stage 1: Use concrete objects and pictorial representations to solve simple addition problems counting all. Introduce counting on using a number track putting the largest number first.

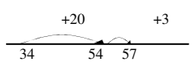
Stage 2: Continue to use practical resources to *support* calculation. Introduce and *model* the use of the numberline. Children begin to use numbered lines to support their own calculations.
Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3 when doing 8+5.



Stage 3: Children will begin to use 'empty number lines' themselves starting with the larger number and counting on irrespective of the order of calculation.
First counting on in tens and ones then helping children to become more efficient by adding the units in one jump (e.g. by using the known fact 4 + 3 = 7).

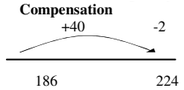


Followed by adding the tens in one jump and the units in one jump.



Children should use their understanding of addition to begin to add fractions with the same denominator $\frac{2}{4} + \frac{2}{4} = \frac{4}{4}$

Stage 4: Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.
Compensation



Children will **begin** to use informal pencil and paper methods (jottings) to support, record and explain mental methods building on existing strategies.

E.g. Partitioning – record mental methods using partitioning .
47 +76 = 76 + 40 +7 = 116 + 7 = 123
Leading to
70 + 6
40 + 7
110+13 = 123

Stage 5:
Secure informal methods leading to the development of more formal written methods.
E.g. Partitioning by adding the most significant digits first, then moving to add the least significant first (Preparation for carrying)

```
  67          267
+ 24        +  85
  80 (60+20)   12 (7+5)
  11 ( 7+ 4)  140 (60+80)
  91          200
              352
```

From this, children will begin to carry below the line.

```
 367          2.84
+ 85        + 1.36
 452          4.20
 11           11
```

**It is important to know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*

Stage 6: Children should extend the carrying method to numbers with at least four digits, including decimals in a range of contexts e.g. money and measurement.

Children should use their understanding of addition to add fractions with the same denominator and fractions with denominators of the same number.
$\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$ $\frac{2}{3} + \frac{3}{4} = \frac{17}{12}$ (or 1 $\frac{5}{12}$)

Stage 7: Children should extend the carrying method to numbers with any number of digits, including decimals and apply their understanding of addition and equivalent fractions to add fractions with different denominators and mixed numbers.
$\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$

Progression in Subtraction

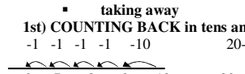
Stage 1: Use concrete objects and pictorial representations to solve simple subtraction problems, involving:-

- **'taking away'** finding how many are left when some are removed
- **'finding the difference'** by using practical objects/resources (e.g. beadstrings, cube towers or number tracks) to make a comparison between the numbers
- **'part/whole' situations** finding how many in a set fit a criteria e.g. 5 cars, 3 are red and the rest are blue. How many are blue?

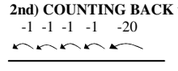
Stage 2: Continue to use practical resources to *support* calculation. Introduce and *model* the use of the numberline. Children begin to use numbered lines to support their own calculations.

- **taking away**

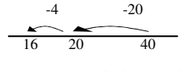
1st) COUNTING BACK in tens and ones



2nd) COUNTING BACK tens in one jump and units/ones in ones



3rd) COUNTING BACK tens in one jump and units/ones in one jump



- **finding the difference**

COUNTING ON to be used when numbers are close together.

Stage 3: Children will continue to use structured number lines leading to empty number lines.

Stage 4: Children will continue to use empty number lines with increasingly large numbers and **will begin to use informal pencil and paper methods (jottings) to support, record and explain mental methods building on existing strategies.**

Partitioning and decomposition *Initially, the children will be taught using examples that do not need the children to regroup.*

```
 89  =  80 + 9
-57      50 + 7
          30 + 2   = 32
```

From this the children will begin to regroup

```
    60
   70  +  11
 -  40  +  6
   20  +  5   = 25
```

Children should demonstrate their place value knowledge lining units up under units, tens under tens, and so on.
They should continue to use an empty number line where the numbers are close together or near to multiples of 10, 100 etc to **count on.** A number line can also be used for subtracting decimals.

Stage 5: Children should be secure in a method for subtraction. Some children may continue confidently working in subtraction on a number line.

Partitioning and decomposition

```
     600    140
   700  +  50  +  14
 -          80  +  6
   600  +  60  +  8  =  668
```

Decomposition

```
  6 14 1          5 14 1
   7 5 4           65.3
 - 8 6           - 8.7
 6 6 8 5 6.6
```

Stage 6: Children should extend the decomposition method to numbers with any number of digits, including decimals in a range of contexts including money and measurement. Number lines and mental methods should also be used, if more efficient than decomposition.

Children should use their understanding of subtraction to subtract fractions with the same denominator and fractions with denominators of the same number.
$\frac{4}{5} - \frac{2}{5} = \frac{2}{5}$ $\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$
**It is important to know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 - 78p.*

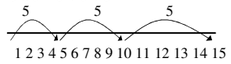
Stage 7: Children should extend the re-grouping method to numbers with any number of digits, including decimals and apply their understanding of subtraction and equivalent fractions to subtract fractions with different denominators and mixed numbers.
$\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

Progression in Multiplication

Stage 1: Children will use concrete objects, pictorial representations and arrays in context, with the support of the teacher, to work on practical problem solving activities involving equal sets or groups. They will begin to make connections between different representations and to count in twos, fives and tens and extend to other patterns.

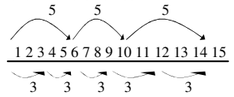
Stage 2: Children will begin to develop their recording using pictures and written descriptions and continue to use a wide range of representations including repeated addition, commutativity, arrays and scaling.
Repeated addition
3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

Repeated addition can be shown easily on a number line:
5 x 3 = 5 + 5 + 5



Commutativity

Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.



Arrays: Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

5x3=15
3x5=15

Scaling: Children will develop an understanding of scaling and comparative language e.g. twice as high, half as long, linked to doubling and halving

Stage 3:
Using every day practical experiences and resources, children will develop an understanding of **multiplicative reasoning** (multiplying, not counting) including through:
Positive Integer **Scaling_**e.g. Find a ribbon that is 4 times as long as the blue ribbon

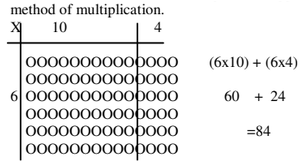
5cm
Use language of scaling e.g. four times as high, eight times as long

Correspondence problems in which m objects are connected to n objects e.g. 3 hats and 4 coats, how many different outfits?

Associativity: (2x3) x4 = 2x(3x4) = 24 or 6x15= 6x(5x3)= (6x5)x3=30x3=90

Partitioning
38 x 5 = (30 x 5) + (8 x 5)
= 150 + 40
= 190

Children will continue to use arrays where appropriate leading into the array (grid) method of multiplication.



Stage 4:
Array method: TU x U and HTU x U
(Short multiplication – multiplication by a single digit)
23 x 8 (Children will approximate first)
23 x 8 is approximately 25 x 8 = 200 x 20 3

8 160 24 160+24=184

Stage 5:) **Array method**: Extended to THTUxU and THTUxTU and also decimals to 3 decimal places e.g. U.t h xU (4.38x7)

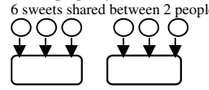
Stage 6: Children should apply their knowledge and understanding of multiplication and use materials and diagrams to multiply proper fractions and mixed numbers by whole numbers e.g. 2/3 x 3 or 2½ x 3

Stage 7: Apply understanding in the use of formal written methods for short and long multiplication and to multiply pairs of proper fractions.

Progression in Division

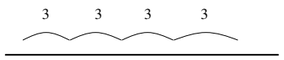
Stage 1: Children will use concrete objects, pictorial representations and arrays in context, with the support of the teacher, to work on practical problem solving activities involving equal sets or groups.
Children will begin to find simple fractions of objects, numbers and quantities.

Stage 2: Children will begin to develop their recording using pictures and written descriptions and continue to use a wide range of representations including:
Sharing equally
6 sweets shared between 2 people, how many do they each get?



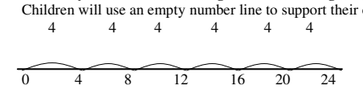
Grouping or arrays:
There are 6 sweets. How many people can have 2 sweets each?
OO
OO
OO

Counting on or back using a number line or bead bar

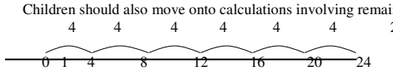


Link fractions to division and find simple fractions (1/3, ¼, ½, ¾) of a length, shape, set of objects or quantity

Stage 3:
Use arrays and link to counting on or back using a number line
Children will use an empty number line to support their calculation.

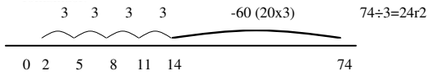


Children should also move onto calculations involving remainders.

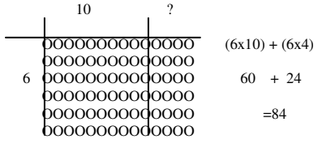


Begin to use division to find unit and non-unit fractions of quantities, understand the effect of dividing a whole number by 10 or 100

Stage 4: Children will develop their use of the reverse array method and number line to add or subtract multiples of the divisor for calculations with or without a remainder.



Reverse array:
84 ÷ 6 =



1?
Leads to 6) 84

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.
Children need to be able to decide what to do after division and round up or down accordingly. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

Stage 5: Children will use their understanding of the array method to use short division for numbers up to THTU ÷ U, interpreting remainders for the context. Children can start to subtract larger multiples of the divisor, e.g. 30x

Stage 6: Children will use their knowledge and understanding of division to use a formal written method to divide numbers up to 4 digits by a 2 digit number. Any remainders should be shown as fractions, i.e. if the children were dividing 32 by 10, the answer should be shown as $3\frac{2}{10}$ which could then be written as $3\frac{1}{5}$ in its lowest terms. They should also apply their knowledge and understanding of division and use materials and diagrams to divide proper fractions by whole numbers. E.g. ⅓ ÷2 = ⅙